TRANSPORT COEFFICIENTS FOR A PARTIALLY IONIZED TWO-TEMPERATURE PLASMA WITH IONS AND NEUTRAL PARTICLES HAVING DIFFERENT MASSES

## L. E. Pekurovskii

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 8, No. 3, pp. 16-24, 1967

The viscosity, heat-conduction, and diffusion coefficients are calculated for a two-temperature three-component plasma composed of ions, neutral particles and electrons when the masses of the ions  $m_i$  and neutral particles  $m_a$  are different. Similar transport coefficients for  $m_i = m_a$  were calculated in [1-3]. The numerical values obtained for the transport coefficients are compared with the values calculated from the formulas of [1-3]. Numerical calculations are carried out for helium with a cesium additive  $(m_i > m_a)$  and for krypton with a lithium additive  $(m_i < m_a)$ .

\$1. The initial system of equations for determining the mass diffusion flux of the  $\alpha$ -component of a plasma  $\mathbf{J}^{\alpha}$ , the viscous stress tensor  $\pi_{ik}{}^{\alpha}$  and the relative heat flux  $\mathbf{h}^{\alpha}$  has the following form [1-3]:

$$\sum_{\beta} (a_{\alpha\beta}J_{i}^{\beta} + b_{\alpha\beta}h_{i}^{\beta}) + \frac{1}{2} \left( a_{\alpha\beta}J_{i}^{\beta} + b_{\alpha\beta}h_{i}^{\beta} \right) + \frac{1}{2} \left( a_{\alpha\beta}J_{i}^{\alpha} + \frac{\partial p_{\alpha}}{\partial x_{i}} + \frac{\partial n_{i}^{\alpha}}{\partial x_{j}^{\alpha}} \right)$$

$$= \frac{1}{2} c_{\alpha\beta}\pi_{ik}^{\alpha} - \frac{1}{2} \left( a_{\alpha\beta}\pi_{ik}^{\alpha} + \frac{1}{2} a_{\alpha\beta}\pi_{ik}^{\alpha} - \frac{1}{2} a_{\alpha\beta}\pi_{ik}^{\alpha} + \frac{1}{2} a_{\alpha\beta}\pi_{ik}^{\alpha} \right)$$

$$= \frac{1}{2} \int_{\beta} c_{\alpha\beta}\pi_{ik}^{\alpha} - \frac{1}{2} \int_{\alpha} c_{\alpha\beta}\pi_{ik}^{\alpha} + \frac{1}{2} \int_{\alpha} c_{\alpha\beta}\pi_{ik}^{\alpha} + \frac{1}{2} \int_{\alpha} c_{\alpha\beta}\pi_{ik}^{\alpha} + \frac{1}{2} \int_{\beta} c_{\alpha\beta}\pi_{ik}^{\alpha} + \frac{1}{2} \int_{\alpha} c_{\alpha\beta}\pi_{ik}^{\alpha} + \frac{1}{2} \int_{\beta} c_{\alpha\beta}\pi_{ik}^{\alpha} + \frac{1}{2} \int_{\beta} c_{\alpha\beta}\pi_{ik}^{\alpha} + \frac{1}{2} \int_{\beta} c_{\alpha\beta}\pi_{ik}^{\alpha} + \frac{1}{2} \int_{\alpha} c_{\alpha\beta}\pi_{ik}^{\alpha} + \frac{1$$

Here the relative velocity, partial pressure, temperature, heat flux, and number of particles per unit volume for the  $\alpha$ -component of the mixture are given by  $\mathbf{w}^{\alpha}$ ,  $\mathbf{p}_{\alpha}$ ,  $\mathbf{T}_{\alpha}$ ,  $\mathbf{q}^{\alpha}$ , and  $\mathbf{n}_{\alpha}$ , respectively,  $\mathbf{u}$  is the mean mass velocity of the gas,  $\omega_{\alpha}$  is the cyclotron frequency of a particle with charge  $\mathbf{e}_{\alpha}$  and mass  $\mathbf{m}_{\alpha}$ ,  $\mathbf{x}$  is a unit vector normal to the direction of the magnetic field, and  $\sigma_{i\mathbf{k}l}$  is the exchange tensor.

The coefficients  $\eta_{\alpha}$  and  $\lambda_{\alpha}$  are associated with the effective collision time  $\tau_{\alpha}$  and  $\tau_{\alpha}^{*}$  by relationships of the form

$$\eta_{\alpha} = \frac{p_{\alpha} \tau_{\alpha}}{2}, \qquad \lambda_{\alpha} = \frac{5p_{\alpha} \tau_{\alpha}^{*}}{2m_{\alpha}}.$$
(1.4)

The quantities  $\tau_{\alpha}^{-1}$  and  $(\tau_{\alpha}^{*})^{-1}$  (reciprocals of the effective collision times) are written as linear combinations of the quantities  $\tau_{\alpha\beta}^{-1}$ , the effective collision frequencies of  $\alpha$  – and  $\beta$ -type particles. General expressions for  $\tau_{\alpha}$ ,  $\tau_{\alpha}^{*}$ , and  $\tau_{\alpha\beta}$  as well as for the coefficients  $a_{\alpha\beta}$ , ...,  $f_{\alpha\beta}$  of the system of equations (1.1)–(1.3) for an ionized gas mixture with an arbitrary number of components are given in the papers cited above.\*

In what follows we consider a three-component mixture composed of ions i, electrons e, and neutral particles a. The temperatures of the heavy components are assumed to be the same  $(T_i = T_a = T)$ . Later, in contrast to [1-3], we assume the masses of the ions and neutral particles to be unequal  $(m_i \neq m_a)$ .

The following conditions are frequently employed in the present paper:

$$\begin{split} &\epsilon = m_e/m_i \ll 1, \quad \delta = m_e/m_a \ll 1, \\ &\epsilon \theta \ll 1, \quad \delta \theta \ll 1 \quad (\theta = T/T_e) \; . \end{split} \tag{1.5}$$

We write the coefficients of the initial system of equations (1.1)-(1.3) for the plasma under consideration:

$$\begin{split} a_{aa} &= -\frac{m_i}{M\tau_{ai}} - \frac{\delta}{\tau_{ae}} \;, \quad a_{ii} = -\frac{m_a}{M\tau_{ia}} - \frac{\varepsilon}{\tau_{ie}} \;, \\ a_{ai} &= \frac{m_a}{M\tau_{ia}} \;, \quad a_{ia} = \frac{m_i}{M\tau_{ai}} \;, \\ a_{ee} &= -\frac{1}{\tau_0} \;, \qquad a_{ei} = \frac{\varepsilon}{\tau_{ie}} \;, \qquad a_{ie} = \frac{1}{\tau_{ei}} \;, \\ a_{ae} &= \frac{1}{\tau_{ea}} \;, \qquad a_{ea} = \frac{\delta}{\tau_{ae}} \;, \end{split} \tag{1.6}$$
 
$$b_{aa} &= -4 \; \frac{\mu}{M} \; \frac{\gamma_i b_1}{\tau_{ai}} - \delta^2 \theta \; \frac{\gamma_a b_2}{\tau_{ae}} \;, \quad b_{ai} = 4 \; \frac{\mu}{M} \; \frac{b_1 \gamma_a}{\tau_{ia}} \;, \\ b_{ae} &= \delta \theta \; \frac{\gamma_a b_2}{\tau_{ea}} \;, \qquad b_{ii} = -4 \; \frac{\mu}{M} \; \frac{\gamma_a b_1}{\tau_{ia}} + 0.6 \varepsilon^2 \theta \; \frac{\gamma_i}{\tau_{ie}} \;, \\ b_{ia} &= 4 \; \frac{\mu}{M} \; \frac{b_1 \gamma_i}{\tau_{ai}} \;, \quad b_{ie} = -0.6 \varepsilon \theta \; \frac{\gamma_i}{\tau_{ei}} \;, \\ b_{ee} &= -\varepsilon \theta \; \frac{\gamma_i}{\tau_1} \;, \quad b_{ea} = \delta^2 \theta \; \frac{b_2 \gamma_a}{\tau_{ae}} \;, \quad b_{ei} = -0.6 \varepsilon^2 \theta \; \frac{\gamma_i}{\tau_{ie}} \;, \end{cases} \tag{1.7}$$

<sup>\*</sup>A full solution of the system of equations (1.1)—(1.3) is set out in detail in the paper of V. A. Polyanskii "Transport phenomena in a multitemperature plasma," doctoral dissertation, Moscow State University, 1965.

$$\begin{split} c_{ea} &= -\delta c_2 \frac{\tau_e}{\tau_{ae}} \,, \qquad c_{ae} &= -\epsilon \left[ c_2 + (1-\theta) \, b_2 \right] \frac{\tau_a}{\tau_{ea}} \,, \\ c_{ai} &= -4 \, \frac{\mu}{M} \, c_1 \frac{\tau_a}{\tau_{ia}} \,, \qquad c_{ia} &= -4 \, \frac{\mu}{M} \, c_1 \frac{\tau_i}{\tau_{ai}} \,, \\ c_{ei} &= -0.4 \epsilon \, \frac{\tau_e}{\tau_{ie}} \,, \qquad c_{ie} &= -\epsilon c_3 \, \frac{\tau_i}{\tau_{ei}} \,, \end{split} \tag{1.8} \\ d_{aa} &= -2.5 \, \frac{4 \mu m_i}{m_a^2} \, \frac{b_1}{\gamma^* \tau_{ai}} \,+ \, 5 \delta \, \frac{1-\theta}{\theta \gamma_a \tau_{ae}} \,, \\ d_{ai} &= 2.5 \, \frac{4 \mu}{m_a} \, \frac{b_1}{\gamma^* \tau_{ia}} \,, \qquad d_{ae} &= \delta \, \frac{d_1}{\tau_a \theta \tau_{ea}} \,, \\ d_{ii} &= -2.5 \, \frac{4 \mu m_a}{m_i^2} \, \frac{b_1}{\gamma^* \tau_{ia}} \,+ \, 5 \epsilon \, \frac{1-\theta}{\theta \tau_i \tau_{ie}} \,, \\ d_{ia} &= 2.5 \, \frac{4 \mu}{m_i} \, \frac{b_1}{\gamma^* \tau_{ai}} \,, \qquad d_{ie} &= \epsilon \, \frac{d_2}{\tau_i \tau_{ei}} \,, \\ d_{ee} &= -2.5 \, \frac{1}{\epsilon \theta \gamma_i \tau_1} \,, \qquad d_{ea} &= 2.5 \, \frac{b_2}{\tau_a \theta \tau_{ae}} \,, \\ d_{ei} &= -1.5 \, \frac{1}{\tau_i \theta \tau_{ie}} \,, \end{cases} \tag{1.9} \end{split}$$

$$\begin{split} f_{ea} &= -\delta f_2 \frac{\tau_{e^*}}{\tau_{ae}}, \quad f_{ei} = -2.78 \frac{\tau_{e^*}}{\tau_{ie}}, \\ f_{ai} &= -8 \frac{\mu m_i}{M^2} f_1 \frac{\tau_{a^*}}{\tau_{ia}}, \quad f_{ae} = -\delta^2 f_3 \frac{\tau_{a^*}}{\tau_{ea}}, \\ f_{ia} &= -8 \frac{\mu m_a}{M^2} f_1 \frac{\tau_{i^*}}{\tau_{i}}, \quad f_{ie} = -\epsilon^2 \theta f_4 \frac{\tau_{i^*}}{\tau_{i}}. \end{split} \tag{1.10}$$

 $f_{aa} = f_{ai} = f_{aa} = 1$ 

Notation:

$$\begin{split} M &= m_a + m_i, \quad \mu = m_a m_i / (m_a + m_i), \\ \gamma_a &= m_a / T_a, \quad \gamma^* = \gamma_a + \gamma_i \;, \\ \tau_0^{-1} &= \tau_{ei}^{-1} + \tau_{ea}^{-1}, \quad \tau_1^{-1} = b_2 \tau_{ea}^{-1} - 0.6 \tau_{ei}^{-1}, \\ b_1 &= 0.25 \, (1.2 C_{ia}^* - 1), \quad b_2 = 1.2 C_{ea}^* - 1 \;, \\ c_1 &= 0.25 \, (1 - 0.6 A_{ai}^*), \\ c_2 &= 1 - 0.6 A_{ae}^*, \quad c_3 = 0.2 \, (3\theta - 1) \;, \\ d_1 &= 2.5\theta^2 b_2 - (1 - \theta) \, (4 A_{ea}^* - 12 C_{ea}^* - 5\theta + 6\theta C_{ea}^*), \\ d_2 &= 3 - 4.5\theta \;, \\ f_1 &= 0.125 \, (5.5 - 1.6 A_{ia}^* - 1.2 B_{ia}^*), \\ f_2 &= 5.5 - 1.6 A_{ea}^* - 1.2 B_{ea}^*, \quad f_4 = 4.5\theta - 1.8 \;, \\ f_3 &= f_2 \theta^2 - \theta (1 - \theta) \, (5 + 1.6 A_{ea}^* + 2.4 B_{ea}^* - 19.2 C_{ea}^* + \\ &+ 2.4 D_{ea}^*) + (1 - \theta)^2 \, (12 C_{eo}^* - 4.8 B_{ea}^* - 2.4 D_{ea}^*) \;. \end{split}$$

Let us now consider the quantities  $\tau_{\alpha}^{-1}$  and  $(\tau^*_{\alpha})^{-1}$ . For the chosen plasma model, they are as follows:

$$\begin{split} \tau_{a}^{-1} &= 0.3\tau_{aa}^{-1} + (\mu/M) \left(1 + 0.6A_{ai}^{*}m_{i}/m_{a}\right)\tau_{ai}^{-1} + \delta\tau_{ae}^{-1}, \\ \tau_{i}^{-1} &= 0.3\tau_{ii}^{-1} + (\mu/M) \left(1 + 0.6A_{ia}^{*}m_{a}/m_{i}\right)\tau_{ia}^{-1} + \epsilon\tau_{ie}^{-1}, \\ \tau_{e}^{-1} &= 0.3\tau_{ee}^{-1} + 0.6A_{ea}^{*}\tau_{ea}^{-1} + 0.6\tau_{ei}^{-1}, \\ \tau_{a}^{*-1} &= \frac{0.4A_{aa}^{*}}{\tau_{aa}} + \frac{\mu m_{i}}{M^{2}\tau_{ai}} \left(\frac{3m_{a}}{m_{i}} + \frac{2.5m_{i}}{m_{a}} + \right. \\ &+ 1.6A_{ai}^{*} - \frac{1.2B_{ai}^{*}m_{i}}{m_{a}}\right) + \frac{3\delta}{\tau_{ae}}, \\ \tau_{a}^{*-1} &= 0.4\tau_{ae}^{-1} + 1.3\tau_{ei}^{-1} + (2.5 - 1.2B_{ea}^{*})\tau_{ea}^{-1}, \end{split}$$

$$\begin{split} \tau_i^{*-1} &= \frac{0.4}{\tau_{ii}} + \frac{\mu m_a}{M^2 \tau_{ia}} \left( \frac{3m_i}{m_a} + \frac{2.5m_a}{m_i} + \right. \\ &+ \left. 1.6 A_{ia}^* - \frac{1.2 B_{ia}^* m_a}{m_i} \right) + \frac{3\varepsilon}{\tau_{ia}} \,. \end{split} \tag{1.12}$$

For the charged particles

$$A_{\alpha\beta}^* = B_{\alpha\beta}^* = 1, \quad C_{\alpha\beta}^* = 1/3,$$

$$D_{\alpha\beta}^* = -1/3, \quad F_{\alpha\beta}^* = -1/9. \quad (1.13)$$

When the interaction between particles differs from a Coulomb interaction, these same quantities have the form

$$\begin{split} A_{\alpha\beta} * &= \frac{\Omega_{\alpha\beta}^{22}}{2\Omega_{\alpha\beta}^{11}} \,, \quad B_{\alpha\beta} * = \frac{5\Omega_{\alpha\beta}^{12} - \Omega_{\alpha\beta}^{13}}{3\Omega_{\alpha\beta}^{11}} \,, \\ C_{\alpha\beta} * &= \frac{\Omega_{\alpha\beta}^{12}}{3\Omega_{\alpha\beta}^{11}} \,, \\ D_{\alpha\beta} * &= \frac{2\Omega_{\alpha\beta}^{23} - 5\Omega_{\alpha\beta}^{22}}{6\Omega_{\alpha\beta}^{11}} \,, \quad F_{\alpha\beta} * = \frac{2\Omega_{\alpha\beta}^{13} - 5\Omega_{\alpha\beta}^{12}}{9\Omega_{\alpha\beta}^{11}} \end{split} \tag{1.14}$$

$$\Omega_{\alpha\beta}^{lr} = \sqrt{\pi} \int_{0}^{\infty} \int_{0}^{\infty} \zeta^{2r+2} \exp(-\zeta^2) g_{\alpha\beta} (1 - \cos^l \chi_{\alpha\beta}) b \, db \, d\zeta . \quad (1.15)$$

Here

$$g_{\alpha\beta} = \left(2 / \gamma_{\alpha\beta}\right)^{1/2} \zeta, \qquad \gamma_{\alpha\beta} = T_{\alpha} / m_{\alpha} + T_{\beta} / m_{\beta}$$

the scattering angle is  $\chi_{\alpha\beta}$ , and the functions  $g_{\alpha\beta}$  and b are determined by the form of the interaction between  $\alpha$ - and  $\beta$ -type particles.

For a model in which the particles are solid spheres, all the quantities  $A_{COB}^*$ ,...,  $F_{COB}^*=1$ .

In the case being considered, the collision times, for charged particles with neutrals and for neutrals with each other, are written in the form

$$\begin{split} \frac{1}{\tau_{aa}} &= \frac{16}{3} \, n_a \left( \frac{T}{\pi m_a} \right)^{1/s} Q_{aa}, \\ \frac{1}{\tau_{ai}} &= \frac{n_i}{n_a} \, \frac{1}{\tau_{ia}}, \frac{1}{\tau_{ae}} = \frac{n_e}{n_a} \, \frac{1}{\tau_{ea}}, \\ \frac{1}{\tau_{ia}} &= \frac{16}{3} \, n_a \left( \frac{T}{2\pi\mu} \right)^{1/s} Q_{ia}, \frac{1}{\tau_{ea}} = \frac{16}{3} \, n_a \left( \frac{T_e}{2\pi m_e} \right)^{1/s} Q_{ea}, \quad (1.16) \\ Q_{\alpha\beta} &= (2\pi\gamma_{\alpha\beta})^{1/s} \Omega_{\alpha\beta} \Pi = \\ &= 2\pi \sqrt{\exp\left(-v^2\right)(1 - \cos\chi_{\alpha\beta})} \, v^{5b} \, db \, dv \; . \end{split}$$

For the interaction of charged particles

$$\frac{1}{\tau_{ee}} = \frac{16}{3} n_e \left( \frac{T_e}{\pi m_e} \right)^{1/2} Q_{ee}, \quad \frac{1}{\tau_{ii}} = \frac{16}{3} n_i \left( \frac{T}{\pi m_i} \right)^{1/2} Q_{ii},$$

$$\frac{1}{\tau_{ie}} = \frac{16}{3} n_e \left( \frac{T_e}{2\pi m_e} \right)^{1/2} Q_{ie}, \quad \frac{1}{\tau_{ei}} = \frac{n_i}{n_e} \frac{1}{\tau_{ie}}, \quad (1.18)$$

$$Q_{\alpha\beta} = \frac{1}{2\pi} (e_{\alpha} e_{\beta} \gamma_{\alpha\beta} / \mu_{\alpha\beta})^2 \ln \Lambda_{\alpha\beta}, \quad (1.19)$$

where  $\Lambda_{\alpha\beta}$  is the Coulomb logarithm.

§2. The general solution of the system of equations (1.2) has the form[2]

$$\pi_{ij}^{a} = -\eta_{\alpha}^{(0)} W_{ij}^{(0)} - \eta_{\alpha}^{(1)} W_{ij}^{(1)} - \eta_{\alpha}^{(2)} W_{ij}^{(2)} +$$

$$+ \eta_{\alpha}^{(3)} W_{ij}^{(3)} + \eta_{\alpha}^{(4)} W_{ij}^{(4)}.$$
(2.1)

Expressions for the second-order tensors  $W_{ij}^{(p)}(p=0,1,2,3,4)$  which are various contractions of the

tensor  $W_{kl}$  with tensor quantities of the type  $\kappa_i \kappa_j \kappa_k \kappa_l$ ,  $\delta_{ij} \kappa_k \kappa_l$ ,  $\sigma_{imk} \kappa_j \kappa_m \kappa_l$ ,  $\sigma_{imk} \sigma_{il} \kappa_m$ , are given in [5].

The viscosity coefficients  $\eta_{\alpha}p$  (p = 0,1,2,3,4) are determined by the relations

$$\begin{split} \eta_{\alpha}^{(0)} &= \sum_{\beta} \frac{\mid c \mid_{\beta\alpha}}{\mid c \mid} \, \eta_{\beta}, \quad \eta_{\alpha}^{(1)} = \sum_{\beta} \frac{\mid c^{*} \mid_{\beta\alpha}}{\mid c^{*} \mid} \, \eta_{\beta}, \\ \eta_{\alpha}^{(2)} &= \sum_{c} \frac{\mid c^{**} \mid_{\beta\alpha}}{\mid c^{**} \mid} \, \eta_{\beta}, \end{split}$$

$$\eta_{\alpha}^{(3)} = \sum_{\beta} \omega_{\beta} \tau_{\beta} \frac{|c^{*}|_{\beta\alpha}}{|c^{*}|} \eta_{\beta}^{(0)}, \ \eta_{\alpha}^{(4)} = \frac{1}{2} \sum_{\beta} \omega_{\beta} \tau_{\beta} \frac{|c^{**}|_{\beta\alpha}}{|c^{**}|} \eta_{\beta}^{(0)} \ (2.2)$$

Here  $\eta_{\alpha}^{(0)}$  is the viscosity coefficient for  $\alpha$ -type particles in the absence of a magnetic field, |c| is the determinant with elements  $c_{\alpha\beta}$ , and  $|c|_{\beta\alpha}$  is the cofactor of the element  $c_{\beta\alpha}$  of this determinant, while  $|c^*|$ ,  $|c^{**}|$  and  $|c^*|_{\beta\alpha}$ ,  $|c^{**}|_{\beta\alpha}$  have similar meanings. Here the elements of the determinants  $|c^*|$  and  $|c^{**}|$  have the form

$$c_{\alpha\beta}^{*} = c_{\alpha\beta} + \frac{|c|_{\beta\alpha}}{|c|} \omega_{\alpha} \tau_{\alpha} \omega_{\beta} \tau_{\beta},$$

$$c_{\alpha\beta}^{**} = c_{\alpha\beta} + \frac{1}{4} \frac{|c|_{\beta\alpha}}{|c|} \omega_{\alpha} \tau_{\alpha} \omega_{\beta} \tau_{\beta}.$$
(2.3)

With the values of the coefficients of  $c_{\alpha\beta}$  given in \$1, the general expressions for the viscosity coefficients (2.2) can be considerably simplified. With an accuracy to quantities on the order of  $\varepsilon^{3/2}$   $\theta^{-1/2}$ , relative to the remaining terms, the determinants  $|c|,|c^*|$  and  $|c^{**}|$  are equal to

$$|c| = \Delta = 1 - c_{ia}c_{at},$$

$$|c^*| = \Delta (1 + \Delta^{-2}\omega_i^2 \tau_i^2) (1 + \omega_e^2 \tau_e^2),$$

$$|c^{**}| = \Delta \left(1 + \frac{\omega_e^2 \tau_e^2}{4}\right) \left(1 + \frac{\omega_i^2 \tau_i^2}{4\Delta^2}\right),$$

$$\Delta = 1 - \left(4 \frac{\mu}{M}\right)^2 c_2^2 \frac{\tau_a \tau_i}{\tau_{ia} \tau_{at}}.$$
(2.4)

The expressions for  $\eta_{\alpha}^{(p)}$  have the same form as formulas (3.3), (3.5), and (3.6) in [2]. However, in the present case the quantities  $\xi_a$  and  $\xi_i$  which appear in these expressions must be replaced by

$$\xi_a = 1 + 4 \frac{\mu}{M} c_2 \frac{\tau_i}{\tau_{ai}}, \quad \xi_i = 1 + 4 \frac{\mu}{M} c_2 \frac{\tau_a}{\tau_{ia}}, \quad (2.5)$$

and the expression for  $\Delta$  is given by the corresponding formula of (2.4).

§3. The solution of the system of equations (1.3) is written as follows:

$$h^{\alpha} = h_r^{\alpha} + h_u^{\alpha} + h_v^{\alpha}. \tag{3.1}$$

The heat flux caused by the temperature gradients of all the components of the mixture is

$$\mathbf{h}_{r}^{\alpha} = -\sum_{\beta} \left[ \lambda_{\alpha\beta}^{\parallel} \nabla_{\parallel} T_{\beta} + \lambda_{\alpha\beta}^{\perp} \nabla_{\perp} T_{\beta} + \lambda_{\alpha\beta}^{\Lambda} \left( \nabla T_{\beta} \times \mathbf{x} \right) \right]. (3.2)$$

The diffusive heat transport is

$$\mathbf{h}_{\mathbf{u}}^{\alpha} = \sum_{\beta} \left[ \mathbf{\mu}_{\alpha\beta}^{\parallel} \mathbf{J}_{\beta}^{\beta} + \mathbf{\mu}_{\alpha\beta}^{\perp} \mathbf{J}_{\perp}^{\beta} + \mathbf{\mu}_{\alpha\beta}^{\Lambda} (\mathbf{J}^{\beta} \times \mathbf{x}) \right]. \quad (3.3)$$

The heat flux due to the viscous momentum transport  $h_{V}^{\alpha}$  will not be considered here, since in the ma-

jority of problems it is of little importance.

Here

$$\lambda_{\alpha\beta}^{\parallel} = \frac{|f|_{\beta\alpha}}{|f|} \lambda_{\beta}, \qquad \mu_{\alpha\beta}^{\parallel} = \sum_{\gamma} \frac{\lambda_{\alpha\gamma}^{\parallel} d_{\gamma\beta} r_{\gamma}^{*}}{\lambda_{\gamma}},$$

$$\lambda_{\alpha\beta}^{\perp} = \frac{|f^{*}|_{\beta\alpha}}{|f^{*}|} \lambda_{\beta}, \qquad \lambda_{\alpha\beta}^{\Lambda} = \sum_{\gamma} \omega_{\gamma} r_{\gamma}^{*} \frac{\lambda_{\alpha\gamma}^{\perp} \lambda_{\gamma\beta}^{\parallel}}{\lambda_{\gamma}}. \qquad (3.4)$$

where  $\tau_{\gamma}^{*}$  are given by formulas (1.12), and the expressions for the coefficients  $\mu_{\alpha\beta}^{-1}$ ,  $\mu_{\alpha\beta}^{\Lambda}$  are obtained from formula (3.4) by replacing the sign || by 1 and  $\Lambda$ , respectively. The elements of the determinant  $|f^{*}|$  are

$$f_{\alpha\beta}^* = f_{\alpha\beta} + \frac{|f|_{\beta\alpha}}{|f|} \omega_{\alpha} \tau_{\alpha}^* \omega_{\beta} \tau_{\beta}^*. \tag{3.5}$$

In the case being considered the determinants |f| and  $|f^*|$  are, with an accuracy to quantities on the order of  $\epsilon^{5/2} \, \theta^{3/2}$ ,

$$|f| = 1 - f_{la}f_{ai} = \Delta_{I},$$

$$|f^{*}| = \Delta_{I} (1 + \omega_{e}^{2} \tau_{e}^{*2}) (1 + \Delta_{I}^{-2} \omega_{i}^{2} \tau_{i}^{*2}) ,$$

$$\Delta_{I} = 1 - \left(4 \frac{\mu}{M}\right)^{3} f_{1}^{2} \frac{\tau_{i}^{*} \tau_{a}^{*}}{\tau_{i} \tau_{i}^{*}}.$$
(3.6)

It follows from (3.2) that the expression for  $h_T^\alpha$  in the general case contains terms which are proportional to the temperature gradients of all components. However, an analysis of the corresponding coefficients shows [2] that in the electron heat flux  $h_T^\alpha$  the basic contribution is from terms which are functions only of  $\nabla T_\alpha$ . Thus

$$\mathbf{h}_{r}^{e} = -\lambda_{e}^{\parallel} \nabla_{\parallel} T_{e} - \lambda_{e}^{\perp} \nabla_{\parallel} T_{e} - \lambda_{e}^{\Lambda} (\nabla T_{e} \times \mathbf{x}). \quad (3.7)$$

Similar estimates enable us, under certain conditions, to omit terms proportional to  $\nabla T_e$  from the expressions for the ion and neutral thermal fluxes. Then

$$\mathbf{h}_{r}^{i,a} = -\lambda_{i,a}^{\parallel} \nabla_{\parallel} T - \lambda_{i,a}^{\perp} \nabla_{\perp} T - \lambda_{i,a}^{\Lambda} (\nabla T \times \mathbf{x}). \quad (3.8)$$

The external form of the coefficients  $\lambda_{\alpha}^{\parallel}$ ,  $\lambda_{\alpha}^{\perp}$ ,  $\lambda_{\alpha}^{\wedge}$  is the same as in formulas (4.4), (4.6), and (4.7) of [2], but the term  $\delta$  which appears there must be replaced by  $\Delta_{1}$  and, moreover

$$\xi_{i}{}^{*} = 1 + 8 \frac{\mu m_{i}}{M^{2}} f_{1} \frac{\tau_{a}{}^{*}}{\tau_{ia}}, \qquad \xi_{a}{}^{*} = 1 + 8 \frac{\mu m_{a}}{M^{2}} f_{1} \frac{\tau_{i}{}^{*}}{\tau_{ai}}.$$

Let us now consider the diffusive heat transport for each plasma component.

As in [2], we represent  $h_{ii}^{\alpha}$  in the form

$$\mathbf{h}_{u}^{\alpha} = \mathbf{h}_{j}^{\alpha} + \mathbf{h}_{s}^{\alpha} . \tag{3.9}$$

Here

$$\begin{split} \mathbf{h}_{j}^{\alpha} &= -\chi_{\alpha}^{\parallel} \mathbf{j}_{\parallel} - \chi_{\alpha}^{\perp} \mathbf{j}_{\perp} - \chi_{\alpha}^{\Lambda} (\mathbf{j} \times \mathbf{x}), \\ \mathbf{j} &= \sum_{\alpha} e_{\alpha} n_{\alpha} \mathbf{u}^{\alpha} \approx e n_{e} (\mathbf{u}^{i} - \mathbf{u}^{e}), \\ \mathbf{h}_{s}^{\alpha} &= -\mu_{\alpha}^{\parallel} \mathbf{s}_{\parallel} - \mu_{\alpha}^{\perp} \mathbf{s}_{\perp} - \mu_{\alpha}^{\Lambda} (\mathbf{s} \times \mathbf{x}), \\ \mathbf{s} &= \mathbf{u}^{i} - \mathbf{u}^{a}. \end{split} \tag{3.10}$$

To calculate the coefficients  $\chi_{\alpha}^{(q)}$  and  $\mu_{\alpha}^{(q)}$   $(q=1, \perp, \vee)$  we transform  $h_{u}^{\alpha(q)}$  to the form\*\*

$$\mathbf{h}_{u}^{\alpha(q)} = -\frac{\mu_{\alpha e}^{(q)} m_{e}}{e} \mathbf{j}_{(q)} - \left[\mu_{\alpha a}^{(q)} \rho_{\alpha} + 2 \left(1 - \alpha^{*}\right) \sum_{\alpha} \lambda_{\alpha \gamma}^{(q)} \xi_{\gamma}\right] \mathbf{s}_{(q)}$$
(3.11)

$$\begin{split} \zeta_{e} &= \varepsilon m_{e} (1 - \theta) \, \tau_{0}^{*-1}, \qquad \zeta_{i} = - \, \theta^{-1} (1 - \theta) \, m_{e} \tau_{ie}^{-1} \,, \\ \zeta_{a} &= - \, \theta^{-1} (1 - \theta) \, m_{e} \tau_{ae}^{-1}, \\ \tau_{0}^{*-1} &= \varepsilon^{-1} (\varepsilon \tau_{ei}^{-1} + \delta \tau_{ea}^{-1}) \,, \\ \alpha^{*} &= \frac{m_{i} n_{i}}{m_{a} n_{a} + m_{i} n_{i}} = \frac{\rho_{i}}{\rho_{a} + \rho_{i}}. \end{split}$$
(3.12)

We write the final expressions for the coefficients  $\chi_O^{(q)}$  and  $\mu_O^{(q)}$ . For diffusive heat transport by electrons

$$\chi_{e}^{\parallel} = -\frac{5}{2} \frac{T_{e}}{e} \frac{\tau_{e}^{*}}{\tau_{1}}, \quad \chi_{e}^{\perp} = \frac{\chi_{e}^{\parallel}}{1 + (\omega_{e} \tau_{e}^{*})^{2}},$$

$$\chi_{e}^{\Lambda} = -\frac{\omega_{e} \tau_{e}^{*}}{1 + (\omega_{e} \tau_{e}^{*})^{2}} \chi_{e}^{\parallel}, \quad (3.13)$$

$$\mu_{e}^{\parallel} = 2.5 p_{e} \tau_{e}^{*} \left[ \frac{b_{2}}{\tau_{ea}} + 2 \varepsilon \frac{(1 - \theta) (1 - \alpha^{*})}{\tau_{ei}} \right],$$

$$\mu_{e}^{\perp} = \frac{\mu_{e}^{\parallel}}{1 + (\omega_{e} \tau_{e}^{*})^{2}}, \quad \mu_{e}^{\Lambda} = -\frac{\omega_{e} \tau_{e}^{*}}{1 + (\omega_{e} \tau_{e}^{*})^{2}} \mu_{e}^{\parallel}. \quad (3.14)$$

The calculations in (3.13) were carried out with an accuracy to terms on the order of  $\varepsilon^{3/2}\theta^{-1/2}$ , and in (3.14) to terms on the order  $\varepsilon^{1/2}$ . For diffusive heat tranport by ions

$$\begin{split} \chi_{i}^{\parallel} &= \frac{\varepsilon^{2} p_{i} \tau_{i}^{*}}{e n_{e} \Delta_{1}} \left[ \frac{\delta_{2}}{\tau_{ie}} + (\xi_{i}^{*} - 1) \frac{m_{i}}{m_{a}} \frac{\delta_{1}}{\theta \tau_{ae}} \right], \\ \chi_{i}^{\perp} &= \\ &= \frac{\varepsilon^{2} p_{i} \tau_{i}^{*}}{e n_{e} \Delta_{1}} \frac{1}{1 + \Delta_{1}^{-2} (\omega_{i} \tau_{i}^{*})^{2}} \left[ \frac{\delta_{2}^{\perp}}{\tau_{ie}} + (\xi_{i}^{*} - 1) \frac{m_{i}}{m_{a}} \frac{\delta_{1}^{\perp}}{\theta \tau_{ae}} \right], \\ \chi_{i}^{\Delta} &= \\ &= \frac{\varepsilon^{2} p_{i} \tau_{i}^{*}}{e n_{e} \Delta_{1}^{2}} \frac{\omega_{i} \tau_{i}^{*}}{1 + \Delta_{1}^{-2} (\omega_{i} \tau_{i}^{*})^{2}} \left[ \frac{\delta_{2}^{\Delta}}{\tau_{ie}} + (\xi_{i}^{*} - 1) \frac{m_{i}}{m_{a}} \frac{\delta_{1}^{\Delta}}{\theta \tau_{ae}} \right], (3.15) \\ \mu_{i}^{\parallel} &= \frac{5 p_{i} \tau_{i}^{*}}{2 \Delta_{1}} \left\{ \left( 2 \frac{m_{a}}{M} \right)_{1}^{2} \frac{b_{1}}{\tau_{ie}} \left[ 1 - \left( 2 \frac{m_{i}}{M} \right)_{1}^{3} f_{1} \frac{\tau_{a}^{*}}{\tau_{ai}} \right] - \right. \end{split}$$

$$\begin{split} &-2\epsilon\frac{1-\theta}{\theta}\left[\frac{(1-\alpha^*)}{\tau_{ie}}-\frac{\alpha^*(\xi_i^*-1)}{\tau_{ae}}\right]\Big\}\,,\\ &\mu_i^{\perp}=\frac{\mu_i^{\parallel}}{1+\Delta_i^{-2}(\omega_i\tau_i^*)^2}\,,\quad \mu_i^{\;\Lambda}=\frac{\omega_i\tau_i^*}{1+\Delta_1^{-2}(\omega_i\tau_i^*)^2}\mu_i^{\parallel}. \end{split} \tag{3.16}$$

In the calculation of the coefficients  $\chi_1^{(q)}$ , all terms appearing in  $\mu_1^{(q)}$  are retained since they all are of the same order. Coefficients  $\mu_1^{(q)}$  were calculated to the accuracy of  $\epsilon^{3/2} \, \theta^{-3/2}$ .

For diffusive heat transport by neutrals

$$\chi_{a} = \frac{\delta^{2} p_{a} \tau_{a}^{*}}{e n_{e} \Delta_{1}} \left[ \frac{\delta_{1}}{\theta \tau_{ae}} + (\xi_{a}^{*} - 1) \frac{m_{a}}{m_{i}} \frac{\delta_{2}}{\tau_{ie}} \right],$$

$$\chi_{a}^{\perp} = \frac{\delta^{2} p_{a} \tau_{a}^{*}}{e n_{e} \Delta_{1}} \times$$

$$\times \frac{1}{1 + \Delta_{1}^{-2} (\omega_{i} \tau_{i}^{*})^{2}} \left[ \frac{\delta_{1}^{\perp}}{\theta \tau_{ae}} + (\xi_{a}^{*} - 1) \frac{m_{a}}{m_{i}} \frac{\delta_{2}^{\perp}}{\tau_{ie}} \right],$$

$$\chi_{a}^{\Lambda} = \frac{\delta^{2} p_{a} \tau_{a}^{*}}{e n_{e} \Delta_{1}^{2}} \times$$

$$\times \frac{\omega_{i} \tau_{i}^{*}}{1 + \Delta_{1}^{-2} (\omega_{i} \tau_{i}^{*})^{2}} \left[ \frac{\delta_{1}^{\Lambda^{*}}}{\theta \tau_{ae}} + (\xi_{a}^{*} - 1) \frac{m_{a}}{m_{i}} \frac{\delta_{2}^{\Lambda}}{\tau_{ie}} \right],$$

$$(3.17)$$

$$\mu_{a}^{\parallel} = \frac{5 p_{a} \tau_{a}^{*}}{2 \Delta_{1}} \left\{ \frac{b_{1}}{\tau_{ai}} \left( 2 \frac{m_{i}}{M} \right)^{2} \left[ 1 - \left( 2 \frac{m_{a}}{M} \right)^{3} f_{1} \frac{\tau_{i}^{*}}{\tau_{ia}} \right] -$$

$$- 2 \frac{\delta (1 - \theta)}{\theta} \left[ \frac{\alpha^{*}}{\tau_{ae}} - \frac{(1 - \alpha^{*})(\xi_{a}^{*} - 1)}{\tau_{ie}} \right] \right\},$$

$$\mu_{a}^{\perp} = \frac{1}{1 + \Delta_{1}^{-2} (\omega_{i} \tau_{i}^{*})^{2}} \times$$

$$\times \left\{ \mu_{a}^{\parallel} - \frac{5 p_{a} \tau_{a}^{*}}{2 \Delta_{1}^{2}} (\omega_{i} \tau_{i}^{*})^{2} \left[ \left( 2 \frac{m_{i}}{M} \right)^{2} \frac{b_{1}}{\tau_{ai}} - 2 \frac{\delta (1 - \theta) \alpha^{*}}{\theta \tau_{ae}} \right] \right\},$$

$$\mu_{a}^{\Lambda} = \frac{\omega_{i} \tau_{i}^{*} \Delta_{1}^{-1}}{1 + \Delta_{1}^{-2} (\omega_{i} \tau_{i}^{*})^{2}} \left\{ \mu_{a}^{\parallel} + \frac{5}{2} p_{a} \tau_{a}^{*} \times$$

$$\times \left[ \left( 2 \frac{m_{i}}{M} \right)^{2} \frac{b_{1}}{\tau_{ai}} - 2 \frac{\delta (1 - \theta) \alpha^{*}}{\theta \tau_{ae}} \right] \right\}.$$
(3.18)

As above, formulas (3.17) contain all the terms appearing in  $\mu_{ae}^{(q)}$ , and expressions (3.18) are calculated with the accuracy of  $\epsilon^{3/2}\theta^{-3/2}$ .

The following symbols are used in formulas (3.15)—(3.18):

$$\begin{split} \delta_1 &= d_1 - 2.5 f_3 \tau_1^{-1} \tau_e^*, & \delta_2 &= d_2 - 2.5 f_4 \tau_1^{-1} \tau_e^*, \\ \delta_1 &= d_1 - 2.5 f_3 \tau_1^{-1} \tau_e^* \varphi_1, & \delta_2 &= d_2 - 2.5 f_4 \tau_1^{-1} \tau_e^* \varphi_1, \\ \delta_1 &= d_1 (1 + \Delta_1^{-1} \omega_4^2 \tau_4^{*2}) - 2.5 f_3 \tau_1^{-1} \tau_e^* \varphi_2, \end{split}$$

Table 1

	1	2		1	2		1	2
n(0)	40.61.10-11	22.11-10-11	$\eta_a^{(0)}$	10.61-10-4	33.39-10-	$\mu_i^{\parallel}$	-93.27·10 <sup>-3</sup>	3.615
$\eta_{e}^{(1)}$	12.97-10-11	13.48-10-11	λe	10 .32 • 1017	5.62-1017	$\mu_a$	8.56	0.953
$n_e^{(2)}$	26.54-10-11	19.03-10-11	$\lambda_e^{\perp}$	7.07-1012	4.93-1017	μe	18.16	27.39
$\eta_e^{(3)}$	19.09-10-11	10.78-10-11	$\lambda_e^{\wedge}$	4.81-1017	1.82-10-7	x <sub>e</sub>	87.13-10-5	10.88-10-8
$\eta_e^{(4)}$	19.49-10-11	7.62-10-11	$\lambda_i^{\parallel}$	28.81 • 1013	94.64-1013	o <sub>0</sub>	1.25-10	67.75-10
$n_i^{(0)}$	26.34-10-	28.50-10-	$\lambda_a^{\parallel}$	62.18-1018	9.35.1017	ro	4.03-10-	1.20-10-4

<sup>\*\*</sup>This expression was obtained by V. A. Polyanskii (see footnote\*).

$$\begin{split} \delta_{1}^{\Lambda} &= d_{1} - 2.5 f_{3} \tau_{1}^{-1} \tau_{e}^{*} \phi_{3} \,, \\ \delta_{2}^{\Lambda} &= d_{2} - 2.5 f_{4} \tau_{1}^{-1} \tau_{e}^{*} \phi_{3} \,, \\ \delta_{1}^{\Lambda^{*}} &= d_{1} \left( 1 - \Delta_{1} \right) + 2.5 f_{3} \tau_{1}^{-1} \tau_{e}^{*} \phi_{4} \,, \end{split} \tag{3.19}$$

$$\phi_{1} &= \frac{1 + \Delta_{1}^{-1} \omega_{i} \tau_{i}^{*} \omega_{e} \tau_{e}^{*}}{1 + (\omega_{e} \tau_{e}^{*})^{2}} \,, \\ \phi_{2} &= \frac{1 - (1 - \Delta_{1}^{-1}) \omega_{i} \tau_{i}^{*} \omega_{e} \tau_{e}^{*} + \Delta_{1}^{-1} (\omega_{i} \tau_{i}^{*})^{2}}{1 + (\omega_{e} \tau_{e}^{*})^{2}} \,, \\ \phi_{3} &= \frac{\Delta_{1} \left( \Delta_{1}^{-1} \omega_{i} \tau_{i}^{*} - \omega_{e}^{*} \tau_{e}^{*} \right)}{\omega_{i} \tau_{i}^{*} \left( 1 + \sigma_{1}^{*} - \omega_{e}^{*} \tau_{e}^{*} \right)} \,, \\ \phi_{4} &= \frac{\omega_{e} \tau_{e}^{*} \left( 1 + \Delta_{1}^{-2} \omega_{i}^{2} \tau_{i}^{*2} \right)}{\omega_{i} \tau_{i}^{*} A_{i}^{-1} \left( 1 + \omega_{e}^{2} \tau_{e}^{*2} \right)} \,. \end{split} \tag{3.20}$$

\$4. When the system of equations (1.1) and (1.3) is solved simultaneously we obtain a result which shows

	Table 2						
,	1	2					
$\eta_i^{(0)}$	29.52-10-	30.18-10-					
λί	18.10-104	14.40-1014					
$\mu_i^{\parallel}$	2.173	3.13					
$\mu_a^{1}$	3.724	62.03					
$r_{\mathbf{e}}$	6.17-10-	6.89-10-					

that the diffusive mass flux of each component is composed of several parts [3]:

$$\mathbf{J}^{\alpha} = \mathbf{J}_{\rho}^{\alpha} + \mathbf{J}_{\tau}^{\alpha} + \mathbf{J}_{v}^{\alpha} + \mathbf{J}_{E}^{\alpha}. \tag{4.1}$$

Expressions for the mass transport caused by the density gradients in the components of the mixture  $J_{p}^{\alpha}$ , the thermal diffusive mass transport  $J_{T}^{\alpha}$ , the mass flux  $J_{T}^{\alpha}$  and mass transfer  $J_{E}^{\alpha}$  by the electromagnetic field are given by the corresponding formulas (1.14), (1.16), and (1.17) of [3].

We assume that anisotropy can be neglected in the transport phenomena ( $\omega_e \tau_e^* \ll 1$ ). Then in our case, with accuracy to quantities on the order of  $\epsilon^{1/2} \theta^{1/2}$  relative to the remaining terms, the elements of determinant  $|a^{(p)}|$ , given for an arbitrary multicomponent plasma (formulas (1.20)), assume the form

$$a_{ai}^{(0)} = \frac{a_1^*}{\alpha \tau_{ai}} + \frac{\delta a_5}{\tau_{ae}}, \quad a_{ea}^{(0)} = \frac{a_0}{\tau_0}, \quad a_{ei}^{(0)} = \frac{a_0}{\tau_0},$$

$$a_{ia}^{(0)} = \frac{a_1^*}{\alpha \tau_{ai}} + \frac{\epsilon a_4}{\tau_{ie}}, \quad a_{ie}^{(0)} = \frac{m_a}{M} \frac{a_1}{\tau_{ia}} + \frac{a_2}{\tau_{ei}},$$

$$a_{ae}^{(0)} = \frac{m_4}{M} \frac{a_1}{\tau_{ai}} + \frac{a_3}{\tau_{ee}}. \quad (4.2)$$

Here

$$\alpha = \frac{n_i}{n_a + n_i}, \quad a_1^* = \frac{m_a + \alpha (m_i - m_a)}{M} a_1,$$

$$a_r = 1 - g_r \quad (r = 0, 1, \dots, 5),$$

$$g_0 = 2.5 \, \tau_1^{-2} \tau_e^* \tau_0, \quad g_2 = -1.5 \, \tau_1^{-1} \tau_e^*,$$

$$g_3 = 2.5 \, b_2 \tau_e^* \tau_1^{-1},$$

$$g_{1} = 5 \frac{b_{1}^{2}}{\Delta_{1}} \left[ \left( 2 \frac{m_{a}}{M} \right)^{3} \frac{\tau_{i}^{*}}{\tau_{ta}} + \left( 2 \frac{m_{i}}{M} \right)^{3} \frac{\tau_{a}^{*}}{\tau_{ai}} - \right.$$

$$- 2 \left( 4 \frac{\mu}{M} \right)^{3} f_{1} \frac{\tau_{a}^{*} \tau_{i}^{*}}{\tau_{ia} \tau_{ai}} \right],$$

$$g_{4} = 1.5 \frac{n_{i}}{n_{e}} \frac{m_{i}}{m_{a}} \frac{\tau_{e}^{*}}{\tau_{2}}, \quad g_{5} = 2.5 \frac{n_{a}}{n_{e}} \frac{b_{2} \tau_{e}^{*}}{\tau_{2}},$$

$$\frac{1}{\tau_{3}} = \frac{b_{2}}{\tau_{ae}} + \frac{m_{a}}{m_{i}} \frac{0.6}{\tau_{ia}}. \quad (4.3)$$

The determinant  $|a^{(0)}|$ , with accuracy to quantities on the order of  $\varepsilon^{1/2} \, \theta^{-3/2}$  relative to the remaining terms, has the form

$$|a^{(0)}| = \frac{a_0^2 a_1^* (1 + \Delta_2)}{2\alpha \tau_0^2 \tau_{ai}},$$

$$\Delta_2 = \frac{2m_e}{\mu} \frac{(1 - 2.5 \tau_e^* \tau_3^{-1}) \tau_0 \tau_{ai}}{a_0 a_1 \tau_{ae} \tau_{ei}}, \quad \frac{1}{\tau_3} = \frac{b_3^2}{\tau_{ea}} + \frac{0.36}{\tau_{ei}}. \quad (4.4)$$

Thus the diffusion coefficients, with accuracy of  $\epsilon^{1/2}\theta^{3/2}$  assume the following form:

$$\begin{split} D_{ae}^{\parallel} &= \frac{T}{m_a} \, \delta^* \left( \frac{m_a}{M} \, \frac{a_1}{\tau_{ia}} + \frac{a_2}{\tau_{ei}} \right), \\ D_{ea}^{\parallel} &= \frac{T_e}{m_e} \, \delta^* \left( \frac{a_1^*}{2 \alpha \tau_{ai}} + \frac{\epsilon a_4}{\tau_{ie}} \right), \\ D_{ai}^{\parallel} &= \frac{T}{m_a} \, \frac{2 \alpha \tau_{ai}}{a_1^* (1 + \Delta_2)}, \quad D_{ia}^{\parallel} &= \frac{T}{m_i} \, \frac{2 \alpha \tau_{ai}}{a_1^* (1 + \Delta_2)}, \\ D_{ei}^{\parallel} &= \frac{T_e}{m_e} \, \delta^* \left( \frac{a_1^*}{2 \alpha \tau_{ai}} + \frac{\delta a_5}{\tau_{ae}} \right), \\ D_{ie}^{\parallel} &= \frac{T}{m_i} \, \delta^* \left( \frac{m_i}{M} \, \frac{a_1}{\tau_{ai}} + \frac{a_3}{\tau_{ea}} \right), \\ D_{a\beta}^{\uparrow} &\approx D_{a\beta}^{\sharp}, \\ D_{\alpha\beta}^{\wedge} \ll D_{\alpha\beta}^{\parallel} \qquad \left( \delta^* = \frac{2 \alpha \tau_{ai} \tau_0}{a_0 a_1^* (1 + \Delta_2)} \right). \end{split} \tag{4.5}$$

The mobilities of the charged particles are (for simplicity assuming  $Zn_i = n_e$ )

$$K_i^{\parallel} = \frac{\mathbf{Z}e}{T} D_{ie}^{\parallel}, \qquad K_e^{\parallel} = \frac{e}{T_e} D_{ei}^{\parallel}. \tag{4.6}$$

Using the expression  $\sigma^{\parallel} = \sum_{\alpha} e_{\alpha} n_{\alpha} K_{\alpha}^{\parallel} [3]$ , we obtain

$$\sigma^{\parallel} = \frac{e^2 n_e \delta^*}{m_e} \left[ \frac{a_1^*}{2\alpha \tau_{et}} + \frac{\varepsilon}{\tau_{es}} \left( \frac{m_i}{m_e} a_5 + \frac{Z n_a}{n_e} a_3 \right) \right]. \quad (4.7)$$

Assuming that  $\varepsilon^{1/2}\theta^{-1/2} \ll 1$ , we have

$$\sigma^{\parallel} = \frac{e^2 n_e \tau_0}{m_e (1 - 2.5 \tau_1^{-2} \tau_e^* \tau_0)}.$$
 (4.8)

\$5. Table 1 gives numerical values for the transport coefficients for the plasma model adopted here with  $m_i \neq m_a$ .

Case (1) considers neutral helium ( $m_a = 6.4 \cdot 10^{-23}$  g) with a cesium additive ( $m_i = 212.6 \cdot 10^{-23}$  g); and so  $m_i > m_a$ . Case (2) considers neutral krypton  $m_a = 133.9 \cdot 10^{-23}$  g) with a lithium additive ( $m_i = 11.1 \cdot 10^{-23}$  g), i.e.,  $m_a > m_i$ . The particles are assumed to be rigid elastic spheres (i.e.,  $Q_{\alpha\beta} = \pi (r_{\alpha} + r_{\beta})^2$ ,  $r_{\alpha}$  and  $r_{\beta}$  are the radii of the spheres) in the collision of a charged particle with a neutral, and also between two neutral particles. Moreover, instead of the diffusion coefficients (4.5) the table gives the conductivity coefficients  $\sigma_0 = n_e e^2 \tau_0 / m_e$  introduced in [4] and also the slipping coefficients

 $r_0 = 2 \delta \tau_{ai} / en_a \tau_{ea}$ , since it is these which appear in the simplified system of equations of motion.

The basic plasma parameters necessary for the calculation were taken as follows:  $B=10^4$  G,  $T=2\cdot 10^{-13}$  erg;  $T_e=10^{-12}$  erg,  $n_a=2\cdot 10^{18}$  cm<sup>-3</sup>, and  $n_e=10^{14}$  cm<sup>-3</sup>. It is interesting to compare the data given in Table 1 with that obtained from the formulas of [2-3], clearly incorrect for the plasma model adopted here  $m_i\neq m_a$ ), but valid for  $m_i=m_a=m$ . In this case the calculations were carried out as if we had no information about the ions, i. e., it was assumed everywhere that  $m=m_a$ , and that  $r_i=r_a$  in the formula  $Q_{\alpha\beta}=\pi \left(r_a+r_i\right)^2$ . The results obtained are given in Table 2. All the remaining transport coefficients which were obtained are the same as in Table 1.

On comparing the tables given above we see that the majority of the transport coefficients could have been calculated from the formulas of [2-3], if the mass m appearing there is everywhere changed to  $m_a$ . However, to calculate the coefficients  $\mu$ ,  $r_0$ ,  $\eta_i$ , and  $\lambda_i$  the formulas obtained in the present paper must be employed, calculations from the formulas of the papers indicated above, when  $m_i \neq m_a$ , may lead to considerable errors (thus, for example, in case (2) calculations for the slipping coefficient may be in error by a factor of almost six).

Finally, the author is grateful to V. V. Gogosov, under whose direction this work was carried out.

## REFERENCES

- 1. M. Ya. Alievskii and V. M. Zhdanov, "Transport equations for nonisothermal multicomponent plasma," PMTF, no. 5, 1963.
- 2. M. Ya. Alievskii, V. M. Zhdanov, and V. A. Polyanskii, "The viscous stress tensor and thermal flux in a two-temperature partially ionized gas," PMTF, no. 3, 1964.
- 3. V. A. Polyanskii, "Diffusion and conductivity in a partially ionized multitemperature gas mixture," PMTF, no. 5, 1964.
- 4. V. M. Korovin, "Possible simplifications for the equations of a two-temperature partially ionized plasma," PMTF [Journal of Applied Mechanics and Technical Physics], no. 6, 1965.
- 5. S. N. Braginskii, "Transport phenomena in a plasma," collection: Problems of Plasma Theory, Vol. 1 [in Russian], Gosatomizdat, 1963.

25 October 1966

Moscow